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# THE EMERGENCE OF SPACE-TIME GRAVITATIONAL PHYSICS AS AN EFFECTIVE THEORY FROM THE $c = 1$ MATRIX MODEL

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## ABSTRACT

We discuss further a recent space-time interpretation of the  $c = 1$  matrix model which retains both sides of the inverted harmonic oscillator potential in the underlying free fermion theory and reproduces the physics of the discrete state moduli of two-dimensional string theory. We show that within this framework the linear tachyon background in flat space arises from the fermi vacuum. We argue that this framework does not suffer from any obvious nonperturbative inconsistency. We also identify and discuss a class of nearly static configurations in the free fermion theory which are interpreted as static metric backgrounds in space-time. These backgrounds are classically absorbing — a beam of tachyons thrown at such a background is only partly reflected back — and are tentatively identified with the eternal back-hole of 2-dimensional string theory.

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# 1 Introduction

One of the most difficult aspects of the  $c = 1$  matrix model has been its connection with the space-time physics of 2-dimensional string theory<sup>1</sup>. In particular, there is no hint in the matrix model of an interpretation as a theory of gravity. Indeed, it is even possible to argue that gravitational effects are absent in this model! However, a careful analysis of this model and its comparison with known facts from perturbative 2-dimensional string theory has revealed a different story. Space-time gravitational physics is now understood to be encoded in the  $c = 1$  matrix model in a subtle and unexpected way and 2-dimensional gravity emerges from it as a low-energy effective theory!

There are two key observations that have made this possible: (i) In the double scaling limit [2] the  $c = 1$  matrix model is equivalent to a theory of nonrelativistic, noninteracting fermions in an inverted harmonic oscillator potential in one space dimension [3]. The semiclassical physics of the matrix model is, therefore, described by a fermi liquid theory. Small fluctuation of the fermi surface around the fermi vacuum are described by a massless scalar excitation [4–6]. The scattering amplitudes for this massless particle are not identical to the tachyon scattering amplitudes of 2-dimensional string theory, but are related to these by momentum-dependent ‘leg-pole’ factors [7–9]. Although this factor is a pure phase in momentum space, in position space it relates the wavefunction of the tachyon of 2-dimensional string theory to the wavefunction of the massless excitation of the matrix model by a nonlocal transformation. It is this nonlocal transformation which gives rise to all of space-time gravitational physics of the string theory, which is otherwise absent in the matrix model [10]. (ii) The symmetric inverted harmonic oscillator potential has two essentially decoupled sides in the semiclassical limit. The abovementioned identification of a massless scalar excitation and its space-time interpretation is based on small fluctuations of the fermi surface on any one side of the potential only (Fig. 1). This would seem to be a sensible thing to do because in the semiclassical limit tunnelling to the other side can be ignored and because “small” classical fluctuations of the fermi surface cannot go over to the other side (unlike the “large” ones which cross the asymptotes, (Fig. 2)). The issue of the other side of the potential is then postponed to a discussion of nonperturbative effects.

This argument, however, cannot be right, if the nonrelativistic fermion theory is the microscopic theory underlying string theory, for the following reason. String theory is a theory of gravity and so for consistency the space-time metric must couple to the energy-momentum tensor of the theory. In particular, it must couple to the total energy of any field configuration in space-time. Now, if the matrix model is the microscopic theory underlying 2-dimensional string theory then it should be possible to map any field configuration in space-time to some field

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<sup>1</sup>For a brief review and a list of original references see ref. [1]

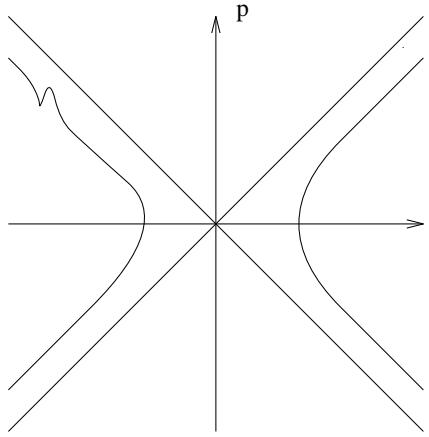


Fig. 1. A small fluctuation of the fermi surface on only one side of the potential.

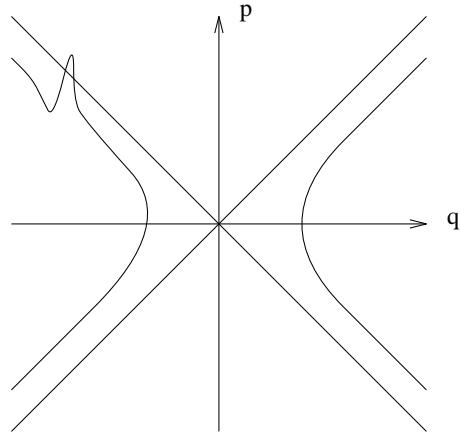


Fig. 2. A large fluctuation of the fermi surface.

configuration in the free fermion theory. Therefore, the energy of this space-time configuration may be computed using the microscopic Hamiltonian. If we decide to retain both sides of the inverted harmonic oscillator potential, then a generic fluctuation will consist of fluctuations of the fermi surface on both sides of the potential (Fig. 3) and so the other side will contribute to the total energy, even in the small-field semiclassical limit when the two sides are otherwise decoupled in the matrix model.

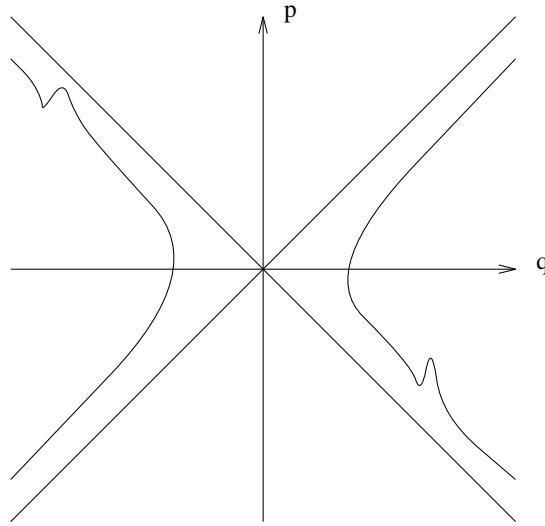


Fig. 3. A small fluctuation of the fermi surface on both sides of the potential.

In the interpretation in (i), based on one side of the potential only [10], the space-time metric is found to couple only to the energy of fermi fluctuation on

that side of the potential, even in the generic case when there is a fluctuation of the fermi surface on the other side also. This is clearly inconsistent with gravitational physics, unless we decide to remove the other side of the potential from the start by introducing a wall (Fig. 4).

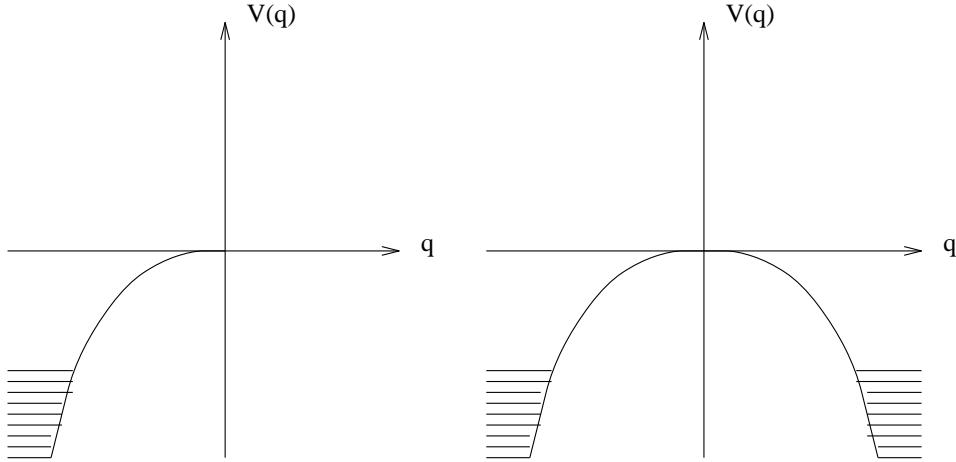


Fig. 4. Filled fermi sea in the modified potential with a wall at  $q=0$ .

Fig. 5. Filled fermi sea in the symmetric inverted harmonic oscillator potential.

Thus, to arrive at a consistent space-time interpretation of this model, we must decide on the fate of the other side of the potential already in the small-field semiclassical limit. The interesting case is the one in which both sides of the potential are retained (Fig. 5) since it is this case that describes the discrete state moduli of 2-dimensional string theory [11]. This possibility arises because in this case at perturbation theory level we have *two* massless scalar fields in the matrix model. One combination of these maps to the tachyon of string theory while the other combination encodes the discrete state moduli.

The organization of this paper is as follows. In the next section we will discuss in some detail the mapping of the matrix model to space-time outlined in (ii) above. We will identify and give the precise relation of the parameters characterising background tachyon and metric perturbations of 2-dimensional string theory with the fields describing the fluctuations of the fermi surface. This is essentially a review of [11]. In Sec. 3 we will show that the linear tachyon background of string theory in flat space originates from the fermi vacuum of the microscopic theory. Together with the results of Sec. 2 this then implies that there is a mapping of *all* the states of the fermi theory to space-time configurations of 2-dimensional string theory, at least in perturbation theory. In Sec. 4 we will identify some configurations in the fermi theory which correspond to essentially static metric backgrounds in their space-time interpretation. The “mass” parameter which characterises these metric backgrounds is continuous. We will further show that these configurations absorb a part of incident tachyon

flux. This makes a tentative identification of these configurations with the eternal black-hole of 2-dimensional string theory very plausible. In Sec. 5 we will discuss a potential nonperturbative inconsistency pointed out by Polchinski [12] and argue that the identification of the weak field expansion parameter in [12], which is one of the crucial elements in Polchinski's argument, may not be correct. Finally we will conclude with some closing remarks in Sec. 6.

## 2 Discrete-state moduli from the free Fermi theory

The nonlocal transformation of the wavefunctions of asymptotic “in” and “out” states of the matrix model, which describes perturbative gravitational physics expected in 2-dimensional string theory, may be expressed in terms of the fermi surface fluctuations as follows:

$$\mathcal{J}_{\text{in}}(x^+) = \int_{-\infty}^{+\infty} d\tau f\left(\left|\frac{\mu}{2}\right|^{1/2} e^{\tau-x^+}\right) \bar{\eta}_{+\text{in}}(\tau), \quad (2.1)$$

$$\mathcal{J}_{\text{out}}(x^-) = \int_{-\infty}^{+\infty} d\tau f\left(\left|\frac{\mu}{2}\right|^{1/2} e^{-\tau+x^-}\right) \bar{\eta}_{-\text{out}}(\tau). \quad (2.2)$$

where  $x^\pm \equiv t \pm x$ ,  $(x, t)$  being space-time labels; “in” and “out” refer to  $t \rightarrow -\infty$  and  $t \rightarrow +\infty$  limits respectively. The space coordinate  $x$  is taken to be large positive keeping  $x^+$  fixed as  $t \rightarrow -\infty$  and  $x^-$  fixed as  $t \rightarrow +\infty$ . The function  $f$  is given by

$$f(\sigma) \equiv \frac{1}{2\sqrt{\pi}} J_0\left(2\left(\frac{2}{\pi}\right)^{\frac{1}{8}} \sqrt{\sigma}\right), \sigma \leq 0 \quad (2.3)$$

The fields  $\bar{\eta}_\pm$  refer to the collective field parametrization of the fluctuations of the fermi surface (relative to the fermi vacuum) which is assumed to have a quadratic profile. For more details on this and on our definitions and notations we refer the reader to ref. [11].

There are two important assumptions that have been made in writing eqns. (2.1) and (2.2). One of these we have already mentioned, namely that these equations assume a quadratic profile for the fluctuations of the fermi surface. This is clearly an undesirable restriction since in general the fluctuations need not have a quadratic profile. Moreover, it is also desirable to write these equations in a form that does not refer to any specific parametrization of the fermi surface since the physics of the tachyon of 2-dimensional string theory should depend only on the profile of the fluctuations and not on any specific parametrization of these. Such a more general form of eqns. (2.1) and (2.2) has been discussed in ref. [13]. Actually, in this reference a more general transformation, which is valid even away from the asymptotic  $t \rightarrow \pm\infty$ ,  $x \rightarrow +\infty$  regions, is obtained from the requirement that the field  $\mathcal{J}(x, t)$  which this transform defines should satisfy the  $\beta$ -function

equation of motion of the tachyon field of 2-dimensional string theory. It turns out that for the purposes of computing tachyon scattering amplitudes only the asymptotic form of the transform is relevant, since the corrections drop out at large positive values of  $x$ . This asymptotic form of the nonlocal transformation is given by

$$\mathcal{J}(x, t) = \int dp dq f(-qe^{-x}) \delta\mathcal{U}(p, q, t) + O(e^{-2x}) \quad (2.4)$$

where  $\delta\mathcal{U}$  denotes a fluctuation of the fermi fluid density  $\mathcal{U}(p, q, t)$  in phase space  $(p, q)$ . We refer the reader to [14] for a detailed account of the phase space formulation of the double-scaled fermion theory and its connection with the collective field theory. For the present purposes, however, the summary given in [11] will be sufficient. We have used the notations and definitions of this reference throughout the present work.

In the asymptotic space-time regions and for a quadratic profile of the fluctuation  $\delta\mathcal{U}$ , the above definition of  $\mathcal{J}(x, t)$  reproduces eqns. (2.1) and (2.2). We stress, however, that this definition of  $\mathcal{J}(x, t)$  allows us to write  $\mathcal{J}_{\text{in}}$  and  $\mathcal{J}_{\text{out}}$  more generally for *any* profile of the phase space density fluctuation. This definition is, in fact, even independent of any parametrization used for the fluctuation, as we shall see later.

The second assumption used in writing eqns. (2.1) and (2.2) is that the fluctuation of the fermi surface on only one side of the potential is relevant, even when there is no wall in the middle, in the semiclassical limit. It is for this reason that the  $q$  integration on the right hand side of eqn. (2.4) is restricted to the region  $-\infty > q \geq -|2\mu|^{1/2}$ . However, as we have argued in the Introduction, in the theory without a wall (i.e. with a symmetrical two-sided inverted harmonic oscillator potential) it is inconsistent to ignore the “other side” of the potential even in the semiclassical limit. We, therefore, need a further generalization of eqn. (2.4) which includes fluctuations of the fermi surface on both sides of the potential. The simplest possibility that suggests itself, because of the symmetry of the potential, is a symmetrical transform. Since the function  $f$  is defined only for positive values of its argument, and since  $q$  has opposite signs on the two sides of the potential, this symmetrical double-sided transform must take the form

$$\mathcal{J}(x, t) = 2^{-1/2} \int dp dq f\left(2^{1/4}|q|e^{-x}\right) \delta\mathcal{U}(p, q, t) + O(e^{-2x}) \quad (2.5)$$

The overall factor of  $2^{-1/2}$  and the extra factor of  $2^{1/4}$  in the argument of  $f$  relative to the expression in eqn. (2.4) have been put there only for later convenience. The  $q$  integration in eqn. (2.5) now extends over the entire real line and so this definition of  $\mathcal{J}(x, t)$  includes density fluctuations of the fermi fluid on both sides of the potential.

The question we now ask is: Does the field  $\mathcal{J}(x, t)$  defined in eqn. (2.5) reproduce the tree-level scattering amplitudes of the tachyon of 2-dimensional string theory? This question is easily answered since eqn. (2.5) allows us to

relate the asymptotic “in” and “out” wavefunctions of the field  $\mathcal{J}(x, t)$ . In this context it is useful to note that, by a change of integration variables and use of the equation of motion of  $\mathcal{U}$ ,  $(\partial_t + p\partial_q + q\partial_p)\mathcal{U}(p, q, t) = 0$ , the right hand side of eqn. (2.5) may be re-expressed in terms of density fluctuation  $\delta\mathcal{U}_0(p, q) \equiv \delta\mathcal{U}(p, q, t_0)$  at some initial time  $t = t_0$ :

$$\mathcal{J}(x, t) = 2^{-1/2} \int dp dq f\left(2^{1/4}|Q(t - t_0)|e^{-x}\right) \delta\mathcal{U}_0(p, q) + O(e^{-x}), \quad (2.6)$$

where  $Q(\tau) \equiv q \cosh \tau + p \sinh \tau$ . Note that the right hand side of eqn. (2.6) does not depend on  $t_0$ . This follows from the equation of motion of  $\mathcal{U}$ .

We may now write down expressions for  $\mathcal{J}_{\text{in}}(x^+)$  (obtained from eqn. (2.6) in the limit  $t \rightarrow -\infty$ , keeping  $x^+$  fixed) and  $\mathcal{J}_{\text{out}}(x^-)(t \rightarrow +\infty, x^- \text{ fixed})$ :

$$\mathcal{J}_{\text{in}}(x^+) = 2^{-1/2} \int dp dq f\left(2^{-3/4} \left| (p - q)e^{t_0} \right| e^{-x^+}\right) \delta\mathcal{U}_0(p, q), \quad (2.7)$$

$$\mathcal{J}_{\text{out}}(x^-) = 2^{-1/2} \int dp dq f\left(2^{-3/4} \left| (p + q)e^{t_0} \right| e^{x^-}\right) \delta\mathcal{U}_0(p, q). \quad (2.8)$$

These expressions for  $\mathcal{J}_{\text{in}}$  and  $\mathcal{J}_{\text{out}}$  do not use any specific form of the profile of the fluctuation of the fermi surface. Moreover, no specific parametrization of the fluctuation of the fermi surface been used in eqns. (2.7) and (2.8). In practice, however, we need to use some parametrization. In the following we will use the familiar collective field parametrization, and to do so we will need to assume a quadratic profile for  $\delta\mathcal{U}_0(p, q)$ . We emphasize that eqns. (2.7) and (2.8) may be used with any other convenient parametrization. We will make a crucial use of this freedom in Sec. 5. We also emphasize that the restriction to a quadratic profile for  $\delta\mathcal{U}_0(p, q)$  does not restrict us to quadratic profiles at all times, as in eqns. (2.1) and (2.2). For sufficiently large fluctuations, in fact, the number of folds changes (in the collective field parametrization) in time. It is, however, clear that in our formalism there cannot be any physical effects associated with such a change since the  $\mathcal{J}_{\text{in}}$  and  $\mathcal{J}_{\text{out}}$  in eqns. (2.7) and (2.8) depend only on the nature of the profile of the initial distribution  $\delta\mathcal{U}_0(p, q)$ <sup>2</sup>.

To proceed further, we will use the independence of the right hand sides of eqns. (2.7) and (2.8) from the parameter  $t_0$  to choose a convenient value for it. In fact, we will take  $t_0 \rightarrow -\infty$ , so that  $\delta\mathcal{U}_0(p, q)$  indeed refers to the initial configuration. Since fluctuations on both sides of the potential are included (Fig. 3) in eqns. (2.7) and (2.8), we need two independent massless fields,  $\bar{\eta}_{+\text{in}}^1$  and  $\bar{\eta}_{+\text{in}}^2$ , similar to the single field  $\bar{\eta}_{+\text{in}}$  which appeared in eqn. (2.1), to describe fluctuations on each of the two sides of the potential. Actually, the following two combinations of these fields appear more naturally in the formalism:

$$\phi(\tau) \equiv (\bar{\eta}_{+\text{in}}^1(\tau) + \bar{\eta}_{+\text{in}}^2(\tau)) / \sqrt{2},$$

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<sup>2</sup>In particular, our formulation makes it clear that the fold radiation discussed in [1] is a singularity associated with the collective field parametrization. There cannot be any physical effect associated with it

$$\Delta(\tau) \equiv \left( \bar{\eta}_{+\text{in}}^1(\tau) - \bar{\eta}_{+\text{in}}^2(\tau) \right) / \sqrt{2}. \quad (2.9)$$

In terms of these fields eqns. (2.7) and (2.8) may be rewritten as

$$\mathcal{J}_{\text{in}}(x^+) = \int_{-\infty}^{+\infty} d\tau \phi(\tau) f \left( \left| \frac{\mu'}{2} \right|^{1/2} e^{\tau-x^+} \right), \quad (2.10)$$

$$\begin{aligned} \mathcal{J}_{\text{out}}(x^-) = & \frac{1}{2} \int_{-\infty}^{+\infty} d\tau \left[ \int_0^{\phi(\tau)+\Delta(\tau)} d\varepsilon f \left( \left| \frac{\mu'}{2} \right|^{1/2} \left( 1 - \frac{\varepsilon}{|\mu'|} \right) e^{-\tau+x^-} \right) \right. \\ & \left. + \int_0^{\phi(\tau)-\Delta(\tau)} d\varepsilon f \left( \left| \frac{\mu'}{2} \right|^{1/2} \left( 1 - \frac{\varepsilon}{|\mu'|} \right) e^{-\tau+x^-} \right) \right]. \end{aligned} \quad (2.11)$$

The details of derivation of these equations may be found in ref. [11]. Here  $\mu' \equiv \sqrt{2}\mu$ .

There are two things that are immediately obvious from eqns. (2.10) and (2.11). The first, as seen from eqn. (2.10), is that it is the *sum* of the fluctuations on the two sides of the potential that maps onto the tachyon of string theory. For  $\Delta(\tau) = 0$  we recover the tachyon scattering amplitudes of string theory in the backgrounds of flat space, linear dilaton and linear tachyon.

The other thing that is clear from eqns. (2.10) and (2.11) is that  $\mathcal{J}_{\text{out}}(x^-)$  does not vanish when  $\varphi(\tau) = 0$  (i.e. when the incoming tachyon field vanishes) but  $\Delta(\tau) \neq 0$ . Classically the only way to interpret this is that for  $\Delta(\tau) \neq 0$  there is a new time-dependent tachyon background present, over and above the linear tachyon background of flat space. This new tachyon background vanishes as  $t \rightarrow -\infty$ , and as  $t \rightarrow +\infty$  it is given by

$$\begin{aligned} (\mathcal{J}_{\text{out}}(x^-))_{\text{background}} = & \frac{1}{2} \int_{-\infty}^{+\infty} d\tau \left[ \int_0^{\Delta(\tau)} d\varepsilon f \left( \left| \frac{\mu'}{2} \right|^{1/2} \left( 1 - \frac{\varepsilon}{|\mu'|} \right) e^{-\tau+x^-} \right) \right. \\ & \left. + (\Delta(\tau) \rightarrow -\Delta(\tau)) \right] \end{aligned} \quad (2.12)$$

Fortunately there is a nontrivial check we can perform to test this interpretation since any background should be reflected in the tachyon scattering amplitudes. In particular, the  $1 \rightarrow 1$  tachyon bulk scattering should see the proposed background in eqn. (2.12). Actually, as it turns out, the  $1 \rightarrow 1$  tachyon bulk scattering amplitude for  $\Delta(\tau) \neq 0$ , which is easily obtained from eqns. (2.10) and (2.11), contains much more than just the amplitude for scattering off the tachyon background identified in eqn. (2.12). A detailed perturbative analysis (for small  $\Delta$ ) at the first nontrivial order in  $\Delta$  has been carried out in ref. [11] for  $1 \rightarrow 1$  tachyon bulk scattering and the leading term in the amplitude at early times ( $x^- \rightarrow -\infty$ ) is compared with what is expected from an analysis of  $\beta$ -function

equations satisfied by tachyon, dilaton and metric of 2-dimensional string theory. In addition to the term expected from scattering off a tachyon background of the form given in eqn. (2.12), the  $1 \rightarrow 1$  bulk scattering amplitude is seen to contain another term which would be expected if there were a background metric given by the line element

$$(ds)^2 = (1 - M e^{-4x})(dt)^2 - (1 + M e^{-4x})(dx)^2, \quad (2.13)$$

where the mass parameter  $M$  equals the energy carried by the field  $\Delta(\tau)$ , i.e.

$$M = \frac{1}{4\pi} \int_{-\infty}^{+\infty} d\tau \Delta^2(\tau). \quad (2.14)$$

We thus see that classically the space-time interpretation of a nonzero value of  $\Delta(\tau)$  is that the tachyon is propagating in space-time background fields. In particular, we have identified the metric perturbation modulus in terms of the microscopic fermion theory variables. We also see that a consistent interpretation of space-time gravitational physics energies in our formalism since the metric couples to the total energy of the system, i.e. to the sum of energies carried by the field  $\phi(\tau)$ , which gives the tachyon fluctuation, and the field  $\Delta(\tau)$ , which gives the backgrounds. It is this sum of energies that equals the total Hamiltonian of the underlying fermion theory. By retaining *both* sides of the potential we have been able to “see” the discrete state moduli of 2-dimensional string theory. It is, therefore, only for this choice that *all* the tree-level physics of 2-dimensional string theory can be extracted from the matrix model.

### 3 The linear tachyon background

Actually there is one aspect of the tree-level physics of 2-dimensional string theory whose origin we have not yet explained in the matrix model. This is the existence of a linear tachyon background in flat space in string theory [15]. To be sure this linear tachyon background can indeed be seen in the matrix model by analysing the  $1 \rightarrow 1$  tachyon bulk scattering amplitude given by eqns. (2.10) and (2.11) for  $\Delta(\tau) = 0$  [10]. However, this does not explain the origin of this background in the matrix model, especially in view of the fact that all the other backgrounds arise from a nonzero value of  $\Delta(\tau)$ . We would now like to show that the linear tachyon background in flat space-time, in fact, arises from the fermi vacuum.

To see this we go back to eqn. (2.4) and examine it more closely. The form of this equation suggests that there might be a mapping of the phase space density itself (and not just its fluctuations) to the unshifted space-time tachyon field  $T(x, t)$ :

$$\begin{aligned} T(x, t) &= e^{-2x} S(x, t), \\ S(x, t) &= 2^{-1/2} \int dp dq f(2^{1/4}|q|e^{-x}) \mathcal{U}(p, q, t) + O(e^{-2x}) \end{aligned} \quad (3.1)$$

If this were the case then eqn. (2.4) would follow from eqn. (3.1) for fluctuations around some “classical” fluid density configuration, which in the present case is the fermi vacuum density  $\mathcal{U}_F(p, q)$ . This ansatz would give rise to a time-independent tachyon background,  $T_F(x)$ , corresponding to the fermi vacuum. In fact this turns out to be precisely the linear tachyon background of string theory in flat space.

Let us verify the above statement. There is a slight subtlety encountered in doing this, namely the right hand side of eqn. (3.1) is divergent for  $\mathcal{U} = \mathcal{U}_F$ . This divergence is associated with the infinite number of fermions present in the double-scaled theory and is a nonuniversal term. To get a finite result one needs to subtract this term from the right hand side of eqn. (3.1) before taking the double-scaling limit. One way to do so is to recognize that the divergent piece does not depend on  $\mu$  and so it may be got rid of by taking a derivative of  $S_F(x)$  with respect to  $\mu$ . This gives

$$\partial_\mu S_F(x) = 2^{-1/2} \int dp dq f(2^{1/4}|q|e^{-x}) \partial_\mu \mathcal{U}_F(p, q) + O(e^{-2x})$$

Since an exact expression for  $\mathcal{U}_F$  is known, the right hand side of this equation can be calculated exactly, but we will here confine ourselves to the semiclassical limit,  $|\mu| \rightarrow \infty$ . In this limit  $\mathcal{U}_F(p, q) = \theta(\mu - \frac{1}{2}(p^2 - q^2))$ , and then we get

$$\partial_\mu S_F(x) = \frac{1}{\sqrt{2\pi}} \left[ 2x + 4\Gamma'(1) - \ln\left(\frac{|\mu|}{\sqrt{\pi}}\right) \right] + O(e^{-x}) \quad (3.2)$$

This shows that  $T_F(x)$  has the form  $(ax + b)e^{-2x}$  to leading order in  $e^{-x}$ . The coefficients  $a$  and  $b$ , which may be obtained from eqn. (3.2), match exactly with those obtained from  $1 \rightarrow 1$  tachyon bulk scattering amplitude in flat space.

We are now in a position to give a string theory interpretation to *all* the states of the underlying microscopic fermi theory. So far we have been treating the field  $\Delta(\tau)$  classically. However, in the quantum theory *both*  $\varphi(\tau)$  and  $\Delta(\tau)$  are fluctuating quantum fields. The above results are obtained by considering those fermi states in which there are a few  $\varphi$  excitations, but in which  $\Delta$  has an expectation value, with small quantum fluctuations. A general state in the microscopic theory, however, has both  $\varphi$  and  $\Delta$  excitations and both the fields undergo quantum fluctuations. As we have seen, the former appear in the string theory as tachyons. The latter may now be seen to give rise to fluctuating quantum fields corresponding to the metric and higher tensor fields of 2-dimensional string theory! The correspondence with the  $W^\pm$  operators of [16] is that the  $W^-$  map onto the states of the fermi theory and  $W^+$  map onto the  $W$ -infinity operators of the fermi theory [17].

## 4 Black hole

In this section we will identify some classical configurations of the field  $\Delta(\tau)$  which seem to have properties of the eternal black hole of 2-dimensional string theory [18].

The Hamiltonian of the fermion theory, in terms of the asymptotic “in” and “out” fields, is given by

$$\begin{aligned} H &= \frac{1}{4\pi} \int_{-\infty}^{+\infty} d\tau [\varphi^2(\tau) + \Delta^2(\tau)] \\ &= \frac{1}{4\pi} \int_{-\infty}^{+\infty} d\tau [\varphi_{\text{out}}^2(\tau) + \Delta_{\text{out}}^2(\tau)]. \end{aligned} \quad (4.1)$$

We have dropped the subscript “in” in keeping with our earlier usage. The “out” fields can be related to the “in” fields in perturbation theory as expansions in string coupling ( $\sim |\mu|^{-1}$ ). We will be interested primarily in the situations in which  $\varphi(\tau) = 0$ . This corresponds to those configurations in space-time in which the “in” tachyon field vanishes. In this case the “out” fields in the microscopic fermion theory have the following perturbative expansions in terms of the only nonvanishing “in” field  $\Delta(\tau)$ :

$$\varphi_{\text{out}}(\tau) = -\left(\frac{1}{2\sqrt{2}|\mu|}\right) \partial_\tau \Delta^2(\tau) + O(\Delta^4/|\mu|^2), \quad (4.2)$$

$$\Delta_{\text{out}}(\tau) = \Delta(\tau) + \frac{1}{12|\mu|^2} \partial_\tau (\partial_\tau - 1) \Delta^3(\tau) + O(\Delta^5/|\mu|^4). \quad (4.3)$$

A check on these expansions is that the second equality of eqn. (4.1) must be satisfied. This is indeed the case, order by order in the string coupling, for the expansions for “out” fields given in eqns. (4.2) and (4.3).

Consider now the class of configurations for which  $\Delta(\tau)$  is a slowly varying function of  $\tau$ . More precisely, let  $L$  be the

scale over which  $\Delta(\tau)$  varies by an appreciable fraction of itself. In the asymptotic past the field  $\varphi$  carries no energy while the energy carried by the field  $\Delta$  scales as  $L$ . In the asymptotic future some of this energy is carried away by the field  $\varphi$ . From eqns. (4.1)–(4.3) we see that, as a fraction of the initial energy, this energy vanishes as  $1/L^2$  for large  $L$ . Combining this with our discussion in Sec. 2, we see that sufficiently slowly varying configurations of  $\Delta(\tau)$  correspond to essentially *static metric backgrounds* in their space-time interpretation. These backgrounds are characterized by a mass parameter, given by the expression in eqn. (2.14), which takes continuous real values. This is large for large  $L$ . Since the only static metric solutions of the  $\beta$ -function equations of 2-dimensional string theory correspond to the eternal black hole, one is led to suspect that the space-time manifestation of the configurations we have identified here might be the eternal black hole.

To test the above hypothesis we will perform a scattering “experiment”. Let us throw a beam of tachyons of frequency  $\omega$  at such a background. A classical black hole must absorb a part of the incoming tachyon flux. For slowly varying  $\Delta(\tau)$ , in the weak field approximation,  $|\Delta(\tau)|/|\mu| \ll 1$ , we can compute the  $1 \rightarrow 1$  tachyon scattering amplitude from the following relation which may be derived from eqns. (2.10) and (2.11):

$$\begin{aligned}\tilde{\mathcal{J}}_{\text{out}}(\omega) &= - \left( \frac{\sqrt{\pi}}{|\mu|} \right)^{i\omega} \left( \frac{\Gamma(i\omega)}{\Gamma(-i\omega)} \right)^2 \Re \tilde{\mathcal{J}}_{\text{in}}(\omega), \\ \Re &= \left[ 1 - \omega(\omega - i) \frac{\Delta_0^2}{4|\mu|^2} + O\left(\frac{\Delta_0^4}{|\mu|^4}\right) \right].\end{aligned}\quad (4.4)$$

Here  $\Delta_0$  denotes the value of  $\Delta(\tau)$  at  $\tau = 0$  and  $\tilde{\mathcal{J}}_{\text{in(out)}}$  is the Fourier transform of  $\mathcal{J}_{\text{in(out)}}$ :

$$\tilde{\mathcal{J}}_{\text{in(out)}}(\omega) \equiv \left( \frac{|\omega|}{\pi} \right)^{1/2} \int_{-\infty}^{+\infty} dx e^{i\omega x} \mathcal{J}_{\text{in(out)}}(x).$$

Note that in writing down eqn. (4.4) we have retained only the leading term in a  $\frac{1}{L}$  expansion. The leading result is independent of  $L$ . So this result is applicable to backgrounds for which the mass parameter  $M \rightarrow \infty$ . Since  $|\Re|^2 < 1$ , we see that the space-time backgrounds under study are absorbing!

Let us summarize: We have identified a class of configurations in the matrix model which have the space-time interpretation of being nearly static (strictly so for  $L \rightarrow \infty$ ) metric backgrounds characterized by large continuous values ( $\rightarrow \infty$  as  $L \rightarrow \infty$ ) of the mass parameter. These metric backgrounds are absorbing — only a part of an incident beam of tachyons is reflected back. Moreover, as argued at the beginning of this section, nearly nothing (strictly so for  $L \rightarrow \infty$ ) ever comes out. It would be interesting to investigate whether one could get at the exact space-time metric for these matrix model backgrounds in the present formalism. There is one interesting question that the reader might ask at this stage: Since the microscopic fermion theory is manifestly unitary, what happens to the tachyons that are absorbed? Fortunately we can answer this question rather easily. We must remember that in the microscopic theory the field  $\Delta$  is as much of a fluctuating quantum field as  $\varphi$  is. We must, therefore, also take the fluctuations of  $\Delta$  around its classical value into account. When we do that we find the following relations between the “in” and “out” modes of the tachyon and the fluctuations of  $\Delta$ , which we shall denote by  $\delta$ :

$$\begin{aligned}\tilde{\mathcal{J}}_{\text{out}}(\omega) &= - \left( \frac{\sqrt{\pi}}{|\mu|} \right)^{\frac{i\omega}{2}} \frac{\Gamma(i\omega)}{\Gamma(-i\omega)} \Re \tilde{\delta}_{\text{in}}(\omega) - \left( \frac{\sqrt{\pi}}{|\mu|} \right)^{i\omega} \left( \frac{\Gamma(i\omega)}{\Gamma(-i\omega)} \right)^2 \Re \tilde{\mathcal{J}}_{\text{in}}(\omega),\end{aligned}\quad (4.5)$$

$$\tilde{\delta}_{\text{out}}(\omega) = - \left( \frac{\sqrt{\pi}}{|\mu|} \right)^{\frac{i\omega}{2}} \frac{\Gamma(i\omega)}{\Gamma(-i\omega)} \Re \tilde{\mathcal{J}}_{\text{in}}(\omega) + \Re \tilde{\delta}_{\text{in}}(\omega). \quad (4.6)$$

Here  $\tilde{\delta}_{\text{in(out)}}(\omega)$  is the Fourier transform of  $\delta(\tau)$ :

$$\tilde{\delta}_{\text{in(out)}}(\omega) = 2\pi|\omega|^{-1/2} \int_{-\infty}^{+\infty} d\tau e^{i\omega\tau} \delta_{\text{in(out)}}(\tau).$$

Also,  $\Re$  is given by the expression in eqn. (4.4) and  $\tilde{\Re} = \frac{\omega\Delta_0}{\sqrt{\pi}|\mu|} + O\left(\frac{\Delta_0^3}{|\mu|^2}\right)$ . Higher order terms in the “in” fields in these equations have been dropped since we are interested only in the  $1 \rightarrow 1$  scattering amplitudes. Eqn. (4.6) shows that an incoming tachyon absorbed by the background metric is actually converted into a  $\delta$  excitation, producing an excited background metric. Eqn. (4.5) shows that these excited background metric states will decay into outgoing tachyons. The amplitudes for these two processes are such that no probability is ever lost and the theory is unitary and time-reversal invariant, as expected.

## 5 Large fluctuations – strong fields

It has been argued by Polchinski [12] that a potential nonperturbative inconsistency exists in the space-time interpretation of the matrix model. His argument concerns large fluctuations of the fermi surface which cross the asymptotes (Fig. 2) and is based on the conservation of  $W$ -infinity charges and an identification of the weak field expansion parameter (WFEP) which allows a perturbative treatment even for large fluctuations. In this section we will argue that the identification of the WFEP in [12] is not correct. We will argue that for large fluctuations in the fermi theory perturbation expansion is not valid in its string theory interpretation.

In the standard collective field parameterization of the fermi surface (assumed to have a quadratic profile) the WFEP can be easily seen to be  $e^{-2\tau}\eta(\tau)/|\mu|$ , where  $\eta(\tau)$  is the asymptotic “in” field that satisfies the equation of a free massless particle. Here  $\tau$  is large positive. This suggests that perturbation theory would be valid even for  $|\eta(\tau)|/|\mu| \gg 1$ , provided we stay at sufficient large  $\tau$  such that the combination appearing in the WFEP is still small. On the string theory side, the  $\beta$ -function equation of motion of the tachyon field,  $\mathcal{J}(x, t)$ , shows that the string theory WFEP is  $e^{-2x}\mathcal{J}(x, t)/|\mu|$ . Thus large values of  $|\mathcal{J}(x, t)|/|\mu|$  do not necessarily invalidate perturbation expansion provided measurements are made at sufficiently large values of  $x$ . This would seem to fit in with the observation of Polchinski made above in the matrix model. An examination of the expression for  $\mathcal{J}_{\text{out}}(x^-)$  in eqn. (2.11), however, shows that a perturbation expansion (assume  $\Delta = 0$ ) is not valid unless we take  $|\eta(\tau)|/|\mu| \ll 1$ . What is the source of this conflict?

There are two crucial points that need to be appreciated to understand the source of the conflict pointed above. One is that large values of  $\mathcal{J}(x, t)/|\mu|$  do not necessarily require large values of  $\eta(\tau)/|\mu|$ . This is most clearly seen in eqns. (2.7) and (2.8) which show that the magnitude of the tachyon field is controlled not just by the *height* of the fluctuation but by the phase space *area* occupied by it. Thus large values of  $\mathcal{J}/|\mu|$  may be obtained from small but well spread out fluctuations of the fermi surface!

The second point has to do with the use of collective field parametrization of the profile of the fermi surface. We have stressed earlier (in Sec. 2) that the physics of the space-time tachyon field should not depend on any specific parametrization of the fermi surface. This was explicitly incorporated in eqns. (2.7) and (2.8) which do not depend on any specific parametrization. So let us use these equations to understand better what is going on.

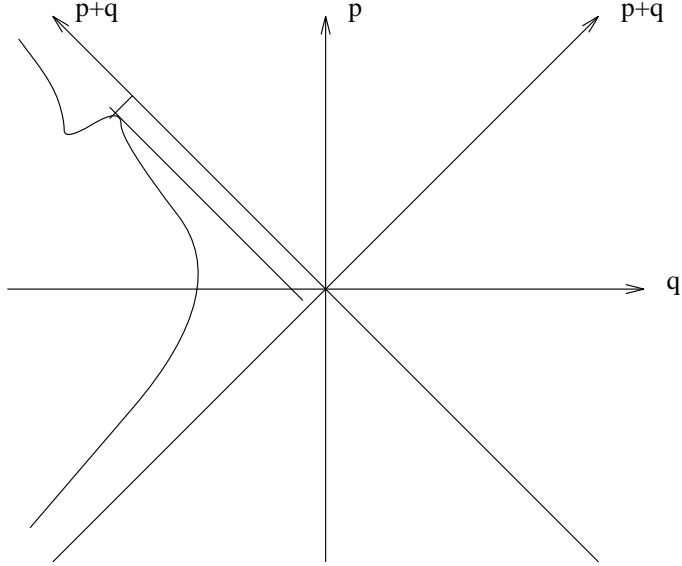


Fig. 6. A parametrization of the fermi surface which uses the "light-cone" variables  $p+q$  and  $p-q$ .

Let us use an alternative parametrization of the fermi surface due to O'Loughlin [19]. In this parametrization, say for the initial configuration of the fermi surface, the distance from the asymptote  $p+q = 0$  is taken to be the field variable,  $\psi(r, t)$ , which depends on the distance  $r$  of the point under consideration from the other asymptote,  $p - q = 0$  (Fig. 6). The WFEP in this case turns out to be  $\eta/|\mu|$ , where  $\eta$  is the field that satisfies the equation of a free massless particle. This is quite different from the WFEP that we got using the collective field parametrization. However, the expressions for  $\mathcal{J}_{\text{in}}$  and  $\mathcal{J}_{\text{out}}$ , as calculated from eqns. (2.7) and (2.8) using this parametrization, work out to be mathematically *identical* to those computed using the collective field parametrization, thereby giving the

same tachyon scattering amplitudes and hence the same space-time physics! This is an explicit verification of the statement that in our formalism the physics of the tachyon of string theory depends only on the profile of the fluctuation of the fermi surface and not on any specific parametrisation used. It is also now clear that the relevant WFEP in the fermi theory, which gives the weak field expansion in string theory, is best read off from eqns. (2.7) and (2.8).

We conclude that the WFEP in the fermi theory is  $\eta/|\mu|$  so that weak fields in the matrix model are identical to small fluctuations. Strong fields correspond to large fluctuations for which perturbation theory breaks down. In the fermi theory this is accompanied by some of the fluid crossing over to the other side, perhaps signalling some new physics. We emphasize that this conclusion is *not* in conflict with the fact that string perturbation expansion is valid even for large values of  $\mathcal{J}/|\mu|$  since, as explained earlier, small values of  $\eta/|\mu|$  already incorporate the possibility of having large values of  $\mathcal{J}/|\mu|$ .

## 6 Concluding remarks

Extracting the space-time physics of 2-dimensional string theory from the  $c = 1$  matrix model has been a long, hard and often frustrating enterprise. It is, therefore, a matter of some satisfaction that we now have a complete mapping from the matrix model to the tree-level physics of string theory, including the discrete state moduli. This correspondence associates flat space-time, linear dilaton and linear tachyon background with the fermi sea of the underlying fermi system. Excited states in the fermi theory are in general associated with tachyons propagating in nontrivial backgrounds of the space-time metric and higher tensor fields. Moreover, as we have argued, there does not seem to be any potential inconsistency in taking the matrix model to define nonperturbative 2-dimensional string theory. What the matrix model might teach us about the nonperturbative structure of string theory remains to be seen, but there are at least two important lessons that have already come through. One is about the nature of microscopic degrees of freedom — these are nothing like strings at all! The other is about the subtle and totally unexpected way in which space-time gravitational physics emerges from the microscopic theory which does not seem to “know” anything about gravitational physics. It is interesting that the currently popular proposal for a microscopic formulation of nonperturbative superstring theory in 10-dimensions [20] shares these two features with the  $c = 1$  matrix model.

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